Depressing the Interferogram Phase Noise Using A Total Variation Approach

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ABSTRACT

Interferometric Synthetic Aperture Radar (InSAR) is used to explore the ground truth by exploiting the phase difference between two synthetic aperture radar images. The phase of an InSAR image pair (called an interferogram) is used to reconstruct a digital terrain model of the imaged area. Typically, the interferogram is noisy due to different effects (thermal, temporal, etc.). In this paper, the interferogram phase noise is characterized by an additive noise model and multiplicative noise model. These models can be employed in a total variation approach to find the optimal approximation of the uncorrupted phase information. The split Bregman iteration approach was used to implement the algorithm. The experimental results show that the proposed approach can robustly and efficiently remove the noise in a numerically stable way. The signal-noise-ratio was used to evaluate the performance of this noise depression approach. From the results, it was found that the additive noise model is more effective at explaining the noise behaviors occurring in an interferogram.

Keywords: Interferogram, InSAR, Total Variation

1. Introduction

Radar interferometry has successfully been applied to reconstruct a digital terrain model (DTM) from two synthetic aperture radar (SAR) images covering the same area with different viewing angles. The two SAR images can be simultaneously collected using two separate antennas or a single antenna at different times. The phase of the two SAR images is called an interferogram, and is modulo $2\pi$ such that the interferogram phase is wrapped and lies in the interval $(-\pi, \pi)$. The DTM can be generated from the wrapped phase. This technique is usually called interferometric synthetic aperture radar (InSAR). The European Remote-Sensing Satellites (ERS) 1 and 2, the shuttle imaging radar SIR-C/X-SAR, TerraSAR-X, and many other SAR systems have successfully employed the technique to generate DTMs with high quality.

The interferogram phase is typically noisy. It is well known that the speckle effects present in an SAR image are the main cause of noise during the creation of an interferogram due to different thermal, temporal, and geometric sources. The phase noise should generally be depressed from the wrapped phase prior to reconstructing the DTM. In doing so, a regular fringe pattern is generated such that the number of irregularities (phase discontinuities) can be significantly reduced. The phase noise can be depressed by applying low-pass filters, such as the Goldstein, boxcar, median, and 2D Gaussian filters (Lee et al., 1998). These filters can effectively depress the noise occurring in the interferogram while losing parts of the spatial resolution. Thus, the filtered results will slightly affect the accuracy of the extracted information (Abdelfattah, 2009).

Lee et al. (1998) proposed an additive noise model to describe the behavior of phase noise in an interferogram. Noise in a given pair of InSAR images is viewed as the interaction of two correlated interference processes (Lee et al., 1998). The additive noise model lets $\phi$ be the corrupted phase, $\phi$ be the uncorrupted phase, and $\eta$ be the noise; then, the relationship can be described as $\phi = \phi + \eta$. Several approaches have been proposed to find solutions based on the additive noise model (Lee et al., 1998; Goldestin and Osher, 2009; Shi et al., 2008). This paper chooses an approach based on a partial...
differentiation equation (PDE) to find an uncorrupted approximation.

A total variation approach has been proposed to find the optimal approximation $\phi$ such that the noise effects can be reduced to a minimum (Rudin et al., 1989). Total variation limits the optimal approximation to always remain in a stable condition, and the differences between the approximation and the uncorrupted phase are expected to be minimal. However, the non-linear properties will increase the time required to find a solution. Recently, the split Bregman iteration scheme was employed to efficiently find a solution (Goldstein and Osher, 2009; Yin et al., 2008). The Bregman iteration scheme is similar to the augmented Lagrangian method for minimizing a convex function, and iteratively solves a sequence of sub-problems of minimizing the sum of the total variation and least squares terms. The split Bregman iterative procedure has an interesting “error-forgetting” property; the difference between the approximation at the $k$th iteration and the optimal solution is bounded by the difference between previous errors during the $k$th and ($k+1$)th iteration, no matter what errors occurred during previous iterations (Yin et al., 2012). This property makes the split Bregman iteration effective at finding solutions to all total variation–based problems.

This paper applies the total variation approach to depress the noise occurring in an interferogram through the split Bregman iteration scheme. In addition, the multiplicative noise model was employed to depress the interferogram phase. The multiplicative noise model, $\varphi = \phi \eta$, has been widely applied to describe the effects of noise in a coherent imaging system. In this paper, the optimal approximation of the multiplicative noise model is obtained by employing the total variation approach. The signal-noise-ratio (SNR) was used to evaluate the performance of the additive and multiplicative noise models at depressing the noise in a given interferogram. This paper is organized as follows. Section 2 explains how to obtain the optimal approximations from the additive and multiplicative noise models based on the total variation approach. An InSAR image pair was employed to generate two noisy interferograms to evaluate the noise depression performance using the introduced approaches. The experimental results are illustrated in Section 3. Finally, discussions and related conclusions are provided in Sections 4 and 5, respectively.

2. Total Variation Approach Based on the Split Bregman Method

In the additive noise model, the problem of depressing the phase noise in a given interferogram can be mathematically stated as follows:

$$\varphi = \phi + \eta,$$

where $\varphi$ is the corrupted phase, $\phi$ is the uncorrupted phase, and $\eta$ is the noise. Based on the facts that the restored solution, $\phi$, is bounded, and that the differences between the restored solution and the given phase reach a minimum, the total variation approach is illustrated as follows (Masnou et al., 1998; Chan et al., 2005; Goldstein and Osher, 2009):

$$E(\phi) = |\nabla \phi| + \frac{\mu}{2} ||\varphi - \phi||_2^2,$$

where $\nabla \phi$ is the gradient of $\phi$ (the term $|\nabla \phi|$ is usually called the total variation),

$$||\varphi - \phi||_2 = \sqrt{\sum_{i=1}^{n} (\varphi_i - \phi_i)^2},$$

and $\mu$ is a constant.

The optimal approximation of $\phi$ can be found by minimizing Eqn. (2) such that $\phi$ is close to $\varphi$. In order to find the optimal approximation ($\phi$) by solving the constrained minimization problem stated in Eqn. (2), Rudin et al. (1998) employed a time evolution scheme. In Eqn. (2), $\phi$ is assumed to be a function of time $t$, and integration over time of the differential equation:

$$\frac{\partial \phi}{\partial t} = \nabla \cdot \left[ \frac{\nabla \phi}{|\nabla \phi|} \right] + \lambda (\varphi - \phi), t > 0,$$

for the initial conditions:

$$\phi = \phi^0 \text{ and } t = 0,$$

in a stable state (Vogel, 1996), where $\phi^0$ is the initial approximation for the solution. The solution can be found as time increases, and the technique used to solve Eqn. (3) is usually called time evolution. A gradient descent scheme based on Newton’s method is usually employed to find the solution. However, this approach generally requires a longer computation time to reach
convergence if the non-linearity and poor conditions are met by applying the descent scheme (Goldstein and Osher, 2009).

Bregman (1967) proposed an iterative approach to determine an optimal approximation from convex functions. Osher et al. (2005) applied an algorithm (called the Bregman iteration method) based on a model developed by Rudin et al. (1992), to remove the noise effects. The model developed by Rudin et al. is called the Rudin–Osher–Fatemi model, and it states that the relationship between the noise and the uncorrupted signal can be shown in an additive way. The Bregman iteration approach can have the value of \( \mu \) remain constant for each iteration. By choosing a suitable value for \( \mu \), the processing speed can be increased if an iterative method, such as the Gauss–Seidel method, is employed. Furthermore, the Bregman iteration approach is error-free: the difference between the \( k \)th solution and the optimal solution is bounded by the difference between the \((k + 1)\)th and \( k \)th errors, independent of the previous errors (Yin et al., 2012). However, the Bregman iteration approach is presumed to be a wide variety of optimization problems.

Goldstein and Osher (2009) proposed the split Bregman method for handling the problem of Eqn. (2) because it can provide an efficient and easy means to depress the phase noise appearing in an interferogram. The split Bregman iteration approach separates the gradient operator in the \( x \) and \( y \) directions, respectively; in other words, \( d_x = \partial \phi / \partial x \) and \( d_y = \partial \phi / \partial y \) are used to replace the term \( |\nabla \phi| \).

Then, Eqn. (2) can be rewritten as follows:

\[
E(\phi) = |d_x| + |d_y| + \frac{\mu}{2} |\phi - \phi_0|^2 + \frac{\lambda}{2} |d_x - \nabla_x \phi - b_x|^2 + \frac{\lambda}{2} |d_y - \nabla_y \phi - b_y|^2,
\]

where \( b_x^k \) and \( b_y^k \) are chosen through Bregman iteration. Equation (6) is solved by applying the iterative minimization approach:

\[
\frac{\partial E}{\partial \phi} = \mu (\phi^{k+1} - \phi) - \lambda \nabla_x^T (d_x^k - \nabla_x \phi - b_x^k) - \lambda \nabla_y^T (d_y^k - \nabla_y \phi - b_y^k) = 0.
\]

Thus, the iterative approach can have \( \phi^{k+1} \) represented by \( \phi^k \), \( d_x^k \), \( d_y^k \), \( b_x^k \), and \( b_y^k \). After rearranging Eqn. (7), the simplified presentation is illustrated as follows:

\[
(\mu d - \lambda \Delta) \phi^{k+1} = \mu \phi + \lambda \nabla_x^T (d_x^k - b_x^k) + \lambda \nabla_y^T (d_y^k - b_y^k),
\]

where \( \Delta \) is defined as a Laplace operator. The Gauss–Seidel method is employed to implement Eqn. (8), which can be represented as follows:

\[
\phi_{i,j}^{k+1} = \frac{\lambda}{\mu + 4\lambda} \left( \phi_{i+1,j}^k + \phi_{i-1,j}^k + \phi_{i,j+1}^k + \phi_{i,j-1}^k + d_{x,i,j}^k - d_{x,i+1,j}^k + d_{x,i,j+1}^k - d_{x,i-1,j}^k - d_{x,i,j-1}^k - d_{x,i,j}^k + b_{x,i,j}^k + b_{x,i,j+1}^k - b_{x,i,j-1}^k + b_{x,i-1,j}^k \right),
\]

The optimal approximation \( \phi \) can be found by applying the split Bregman iteration approach, and the implementation of the algorithm is summarized as follows:

**STEP 3.** Let \( d_x^0 = \text{shrink}(\nabla_x \phi^0 + b_x^0, 1/\lambda) \).

**STEP 4.** Let \( d_y^0 = \text{shrink}(\nabla_y \phi^0 + b_y^0, 1/\lambda) \).

**STEP 5.** Let \( b_x^0 = b_x^0 + (\nabla_x \phi^0 - d_x^0) \).

**STEP 6.** Check the convergence by calculating, \( |\phi^1 - \phi^0|^2 \). If the difference is less than the pre-defined threshold, then the procedure ends.

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Otherwise, the procedure will repeat from STEP 2 to STEP 6.

Donoho (1993) proposed the shrink operator based on statistical observations of noise behavior. It has been widely applied to reduce the computational complexity. Goldstein employed the technique to increase the processing speed. The shrink operator is defined as follows:

\[
\text{shrink}(x, r) = \begin{cases} 
\frac{x}{|\mathbf{x}|} \max(|x| - r, 0) & x \in [r, \infty) \\
0 & x \in (-r, r) \\
(x - r) & x \in (-\infty, -r) 
\end{cases} \tag{10}
\]

For the multiplicative model, \( \varphi = \phi \eta \), the logarithm operator is applied to the model such that the multiplicative relationships can be replaced by the additive relationship, \( \log \varphi = \log \phi + \log \eta \). Therefore, Eqn. (2) can be rewritten as follows:

\[
E(\log \phi) = |\nabla(\log \phi)| + \frac{\mu}{2} \|\log \varphi - \log \phi\|^2_2 \tag{11}
\]

By letting \( w = \log \phi \) and \( q = \log \varphi \), Eqn. (11) can be written as:

\[
E(w) = |\nabla w| + \frac{\mu}{2} \|q - w\|^2_2 \tag{12}
\]

Hence, the split Bregman iteration approach can still be employed to depress the noise behavior in the multiplicative model with few modifications.

### 3. Experimental Results

A pair of InSAR is employed to evaluate the performance of the proposed approach; one is collected by ERS-1 on August 01, 1995, and another one is collected by ERS-2 on August 02, 1995. The information of those two images is given in Table 1. The geographic location indicated by white square is illustrated in Fig.1, and the locations of the two test areas are shown in ERS-1 and labeled A and B.

With precise orbital information provided by Delft University and co-registration of the given InSAR pair, the coherence images and the interferogram can be generated. In this paper, two sub-regions (labeled A and B) were extracted, and their corresponding coherence images and interferograms were generated. Each sub-region contains 5000×2400 pixels. The additive and multiplicative noise models implemented with the split Bregman iteration approach were employed to depress the noise appearing in the interferograms. In Fig. 2 and Fig. 3, the coherence images and interferograms are illustrated, respectively. The noise shown in both interferograms was depressed by applying a Goldstein filter (Goldstein et al., 1998). The resulting filtered interferograms are illustrated in Fig. 4. The SNR was used to evaluate the performance of the interferogram phase noise depression, and the definition is given as follows:

\[
\text{SNR} = 20 \log_{10} \left( \frac{\|\phi\|_2}{\|\eta\|_2} \right), \tag{13}
\]

where in the additive noise model, \( \eta = \varphi - \phi \), and in the multiplicative noise model, \( \eta = \varphi / \phi \).

<table>
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<th>Data Type</th>
<th>Date</th>
<th>Track</th>
<th>Orbit</th>
<th>Bperp(m)</th>
<th>Master/Slave</th>
<th>RowsxColumns</th>
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<td>21159</td>
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<td>ERS-2</td>
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<td>129</td>
<td>1486</td>
<td>0</td>
<td>Master</td>
<td>29650x4903</td>
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</tbody>
</table>
a. The geographic location of the test area, shown in white-red square box
b. The given ERS-1 SAR illustrates the test areas

Figure 1 The related informations for the test regions.

Figure 2. The coherence image and interferogram of sub-region A generated from an InSAR pair.

Figure 3. The coherence image and interferogram of sub-region B generated from an InSAR pair
By setting several parameters (patch size $32 \times 32$, overlap 8, alpha 0.5, and kernel $\{1,2,3,2,1\}$) in the Goldstein filter, the SNR for the regions A and B were found to be 1.6244 and 1.3497, respectively. In Goldstein filter, the patch is a small part of the given interferogram and are overlapped to prevent those discontinuities illustrated on the boundaries. The alpha is a parameter related to the filter performance; for a big alpha, the filtering results are significant. In Fig. 4, there is noise present in the filtered interferograms, and more sophisticated approaches are needed to deal with the noisy interferograms.

The additive and multiplicative noise models were adopted to describe the noise behavior of the interferograms, and the split Bregman iteration approach was implemented to find the optimal approximation. With the parameters $\mu = 1$ and $\lambda = 100$, the optimal approximation can be found, and, for sub-region A, the processed results of the additive noise model are illustrated in Fig. 5.

With the split Bregman iteration approach, an optimal approximation can be found based on the total variation approach shown in Eqn. (5). The relationship between the number of iterations and the energy defined in Eqn. (5) is presented in Fig. 6. It can be seen that after 10 iterations, the value of the energy decreases to a stable condition. This indicates that this approach can maintain numerical stability. After a few iterations, the error defined by the differences between two iterated approximations $\phi^k$ and $\phi^{k+1}$, $\|\phi^{k+1} - \phi^k\|_2$, is shown in Fig. 7. It can be seen that the error quickly drops to a small value. The value of the SNR was around 5.44 after 100 iterations.

![Image](image1.png)

**Figure 4.** The noisy interferograms filtered by employing the Goldstein filter

![Image](image2.png)

**Figure 5.** The iterated results for sub-region A after applying the split Bregman iteration approach in the additive noise model
Similarly, the multiplicative noise model implemented by the split Bregman iteration approach was employed to depress the noise shown in the interferogram of sub-region A. With the parameters $\mu = 1$ and $\lambda = 100$, the uncorrupted approximation can be found after a few iterations. The processed results are shown in Fig. 8. The relationships between the number of iterations and different factors (energy, error, and SNR) are illustrated in Figs. 9 and 10. The SNR of the optimal approximation was 3.09 after 100 iterations.

A comparison of the processed results after employing the additive and multiplicative noise models to find the optimal approximation based on the split Bregman iteration shows that all the curves shown in the figures are similar. Furthermore, the value of the SNR under the same parameters seems to indicate that the additive noise model is more suitable to depressing the noise appearing in the interferogram of sub-region A than the multiplicative noise model. The same approaches were applied to sub-region B to evaluate their performance.

Sub-region B was extracted from the same InSAR pair, and its size was also 5000×2400 pixels. The additive and multiplicative noise models implemented by the split Bregman iteration approach were applied on sub-region B. The same parameters $\mu = 1$ and $\lambda = 100$ were employed in this iteration approach. The processed results after applying the additive noise model are shown in Fig. 11, and the different ways to evaluate the convergence are shown in Fig. 12. The value of the SNR was around 5.44 after 100 iterations. Similarly, the processed results after applying the multiplicative noise model implemented by the split Bregman iteration approach are shown in Fig. 13 and Fig. 14. The SNR of the optimal approximation was 3.08 after 100 iterations.
Figure 9. The relationship between the number of iterations and the energy defined by (5)

![Energy vs Iterations](image)

Figure 10. The relationship between the error defined by $|\phi^1 - \phi^0|_2$ and the number of iterations

![Error vs Iterations](image)

Figure 11. The uncorrupted approximations at different iterations generated by applying the additive noise model implemented by the split Bregman iteration approach

Sub-region B was extracted from the same InSAR pair, and its size was also 5000×2400 pixels. The additive and multiplicative noise models implemented by the split Bregman iteration approach were applied on sub-region B. The same parameters $\mu = 1$ and $\lambda = 100$ were employed in this iteration approach. The processed results after applying the additive noise model are shown in Fig. 11, and the different ways to evaluate the convergence are shown in Fig. 12. The value of the SNR was around 5.44 after 100 iterations. Similarly, the processed results after applying the multiplicative noise model implemented by the split Bregman iteration approach are shown in Fig. 13 and Fig. 14. The SNR of the optimal approximation was 3.08 after 100 iterations.
(a). The relationships between iterations and the energy are illustrated.

(b). The relationships between the errors and iterations are shown.

Figure 12. The different ways to evaluate the convergence by applying the additive noise model implemented by the split Bregman iteration approach.

Figure 13. The uncorrupted approximations at different iterations generated by applying the multiplicative noise model implemented by the split Bregman iteration approach.
Discussions

Depressing the noise shown in an interferogram is a crucial step in generating a DTM from a given InSAR pair. The Goldstein filter is widely applied in different InSAR software programs to depress the phase noise shown in interferograms, such as the Delft object-oriented radar interferometric software, Doris. From the experiment, the values of the SNR for regions A and B were found to be 1.6244 and 1.3497, respectively. In this paper, the additive and multiplicative noise models implemented by applying the split Bregman iteration were applied to depress the noise presented in the interferograms of sub-regions A and B. From the experimental results, it was determined that regardless of the model employed to depress the phase noise, the proposed approaches can efficiently depress the phase noise shown in the interferograms. With the same parameters, the relationships between the number of iterations and the energy, and between the error and the SNR, are similar to those between the additive and multiplicative noise models. Furthermore, the additive noise model is more effective at depressing phase noise than the multiplicative noise model. In this section, the performance is evaluated after setting different values for the parameters $\mu$ and $\lambda$. To simplify the evaluation, these procedures were only applied on the interferogram of sub-region A.

The two parameters $\mu$ and $\lambda$ were first set to implement the split Bregman iteration approach (Goldstein and Osher., 2009). In this paper, different combinations of $\mu$ and $\lambda$ values were employed in the approach. In this paper, different combinations of $\mu$ and $\lambda$ values were employed in the approach. Firstly, for fixed $\mu$ and different $\lambda$ values, the processed results are illustrated in Fig. 15. Similarly, for fixed $\lambda$ and different $\mu$ values, the processed results are illustrated in Fig. 16. The values of $\mu$ and $\lambda$ play an important role in the performance of the split Bregman iteration approach.
role in the split Bregman iteration approach; these two values weigh the third, fourth, and fifth terms in Eqn. (6), respectively. From the processed results, it can be observed that with a large $\mu$ value, the depressing noise effects are not obvious, and that with a large $\lambda$ value, the details will be maintained, such as edges.

The third term, $|\phi - \phi|^2$, is established to determine the optimal approximation in Eqn. (6). As $\mu$ increases, the third term dominates Eqn. (6), such that the optimal approximation is close to the solution according to the least squares principle. Conversely, as $\lambda$ increases, the fourth and fifth terms, $|d_x - \nabla_x \phi - b^k_x|^2$ and $|d_{xy} - \nabla_y \phi - b^k_y|^2$, dominate Eqn. (6), such that detailed information in the processed results may be retained. The third term acts like a smooth filter that allows low-frequency information to pass but filters out some details and noise.

Figure 15. The processed results after applying different values of $\mu$ and $\lambda$ (Left: $\mu = 1, \lambda = 1$, Center: $\mu = 1, \lambda = 10$, Right: $\mu = 1, \lambda = 100$)

Figure 16. The processed results after applying different values of $\mu$ and $\lambda$ (Left: $\mu = 1, \lambda = 10$, Center: $\mu = 10, \lambda = 10$, Right: $\mu = 100, \lambda = 10$)
The values of $\mu$ and $\lambda$ affect the defined energy shown in Eqn. (6), and the results are illustrated in Fig.17 and Fig 18, respectively. With a fixed $\mu$ and varying $\lambda$ values, it can be observed that a small $\lambda$ value will produce a high energy. From the natural condition, the stable condition always corresponds with a low energy. Hence, when $\mu = 1$ and $\lambda = 100$ were employed in the split Bregman iteration, the defined energy was small in comparison with the other parameter settings; in other words, the solution will be in a stable condition. However, with a fixed $\lambda$ and varying $\mu$ values, the processed results are different. Therefore, the $\mu$ parameter is not a major factor affecting the defined energy; when the value of $\mu$ increases, the energy will not increase.

The interferogram phase usually contains noise, and the Goldstein filter is widely used to depress the effects of noise. In the filtered results, there are still some regions in which noise cannot be removed. In this paper, the additive and multiplicative noise models implemented with the split Bregman iteration approach were employed to depress the noise in the interferogram phase. Based on the SNR values, it was found that the additive noise model has higher SNR values than the multiplicative noise model. Noise occurring in the interferogram is viewed as the interaction of two correlated and coherent interference processes. The complex correlation coefficient can generally be separated into two parts, the magnitude ($p$) and the phase ($\theta$). The phase distribution $\varphi$ shown in the interferograms is symmetric about $\theta$, and $\theta$ is the mean with modulus $2\pi$ (Lee et al., 1998). Based on the statistical properties of the phase distribution, the standard deviation is independent of $\theta$. Consequently, the additive noise model is suitable for describing the noise behavior shown in the interferogram phase (Lee et al., 1998). In this paper, according to the SNR values, the additive noise model appears to be more effective at depressing the noise effects in the interferograms than the multiplicative noise model. Furthermore, by visually examining the processed results, it is obvious that the additive noise model can also retain more detailed information than the multiplicative noise model.

![Figure 17. The relationship between the defined energy and the number of iterations with fixed $\mu$ and different $\lambda$ values](image1.png)

![Figure 18. The relationship between the defined energy and the number of iterations with fixed $\lambda$ and different $\mu$ values](image2.png)
The split Bregman iteration approach was used to implement the additive and multiplicative noise models based on the principles of total variation. Typically, the Gauss–Newton method is employed on the model based on the total variation approach. However, the split Bregman iteration approach can quickly achieve convergence in a few iterations. Two criteria are defined to halt the algorithm: error and energy. The experimental results show that the algorithm requires less than 5 iterations before the error is less than a predefined threshold and can halt. However, if the energy is less than a predefined threshold, then it always requires more than 20 iterations to achieve convergence. No matter which approach is chosen to halt the procedure, the split Bregman iteration approach provides an efficient and numerically stable way to find the optimal approximation of the uncorrupted phase information.

The values of $\mu$ and $\lambda$ will significantly affect the processed results. The value of $\mu$ should be kept small, and the value of $\lambda$ should be kept large to obtain good-quality processed results. In these experiments, the values of $\mu$ and $\lambda$ were found using a trial-and-error approach. Typically, $\mu = 1$ and $\lambda = 10$ will be a good choice to begin the implementation. Then, the values of $\lambda$ can be changed with a fixed $\mu$ value. However, the value of $\lambda$ should be limited because when the value of $\lambda$ is large, the energy is dominated by the two differential terms in the $x$ and $y$ directions, such that the norms of the differences between the noisy phase and the uncorrupted phase information will not play a role in the whole processing procedure. The total variation approach limits the optimal approximation such that optimal approximation is bounded and the uncorrupted phase information can be extracted from a stable system rather than from a disorganized and chaotic system.

5. Conclusions

The noise present in an interferogram usually causes difficulties in restoring the digital terrain model. It is therefore necessary to depress the noise effects. The additive and multiplicative noise models are typically employed to depress the noise occurring in a given interferogram. Different algorithms have been proposed to deal with this problem. The total variation approach implemented with the split Bregman iteration approach provides an efficient and numerically stable means of depressing the noise effects. The additive noise model is suitable for describing the noise behaviors appearing in the interferogram based on the values of their signal-to-noise ratio.

References


利用 Total Variation 抑制干涉條紋圖中相位雜訊

黃怡碩

摘 要

合成孔徑雷達影像藉由兩幅合成孔徑雷達影像的相位資訊，形成干涉條紋圖，利用干涉原理，則所生成的干涉條紋圖可生成所拍攝區域的數值地形。由於，受到合成孔徑雷達影像本身雜訊的影響，使得產生的干涉條紋圖亦具有雜訊。本文中將干涉條文圖中相位雜訊分別考慮期在相加及相乘模組的作用，利用 Total Variation 處理模組，進行雜訊抑制；除此之外，利用訊雜比(亦即訊號與雜訊的比值)，進行雜訊抑制成效的評比。本文引進 Split Bregman 處理模組處理 Total Variation 處理模組中相加與相乘的雜訊模式；因 Split Bregman 處理模組其為一解偏微分的處理模組，利用疊代的方式，可以快速地獲得一組數值穩定的最佳解，使得最佳未受雜訊干擾的資訊能被獲得。從實驗結果發現，以相加模組解釋干涉條紋圖中相位雜訊的影響是較為適宜，此一結論與前人所做研究結論相符且其數值穩定度高。

關鍵詞：干涉、相位條紋圖、雜訊